Reg. No. : | | | | | | | | | | | |

R 3422

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fourth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1252 — PROBABILITY AND QUEUEING THEORY

(Common to B.E. Part-Time Third Semester Regulation 2005)

Time : Three hours   Maximum : 100 marks

Use of statistical table is permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the law of total probability and state under which situation it could be used.

2. Suppose that a bus arrives at a station every day between 10.00 a.m. and 10.30 a.m., at random. Let X be the arrival time; find the distribution function of X and sketch its graph.

3. Sharon and Ann play a series of backgammon games until one of them wins five games. Suppose that the games are independent and the probability that Sharon win a game is 0.58.

   (a) Find the probability that the series ends in 7 games.

   (b) If the series ends in 7 games, what is the probability that Sharon wins?

4. Define exponential random variable. Give an example.

5. For $\lambda > 0$, let $F(x,y) = \begin{cases} 1 - \lambda e^{-\lambda(x+y)} & \text{if } x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$

   Check whether F can be the joint probability distribution function of two random variables X and Y.

6. The life time of a TV tube (in years) is an exponential random variable with mean 10. What is the probability that the average lifetime of a random sample of 36 TV tubes is atleast 10.5?

7. Suppose that people immigrate into a territory at a Poisson rate $\lambda = 1$ per day.

   (a) What is the expected time until the 10th immigrant arrives?

   (b) What is the probability that the elapsed time between the 10th and the 11th arrival exceeds 2 days?

8. At an intersection, a working traffic light will be out of order the next day with probability 0.07, and an out-of-order traffic light will be working the next day with probability 0.88. Let $X_n = 1$ if on day 'n' the traffic light will work; $X_n = 0$ if on day 'n' the traffic light will not work.

   Is $\{X_n; n = 0,1,2,...\}$ a Markov chain? If so, write the transition probability matrix.

9. Suppose that customers arrive at a Poisson rate of one per every 12 minutes, and that the service time is exponential at a rate of one service per 8 mins.

   (a) What is the average no. of customers in the system?

   (b) What is the average time of a customer spends in the system?

10. What do you mean by transient state and steady state queueing system?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A box contains 7 red and 13 blue balls. Two balls are selected at random and are discarded without their colours being seen. If a third ball is drawn randomly and observed to be red, what is the probability that both of the discarded balls were blue? (8)

   (ii) The sales of a convenience store on a randomly selected day are X thousand dollars, where X is a random variable with a distribution function of the following form :

$$F(x) = \begin{cases} 0; & x < 0 \\ \dfrac{x^2}{2}; & 0 \le x < 1 \\ k(4x - x^2); & 1 \le x < 2 \\ 1; & x \ge 2 \end{cases}$$

Suppose that this convenience store's total sales on any given day are less than \$ 2000.

(1) Find the value of k.

(2) Let A and B be the events that tomorrow the store's total sales are between 500 and 1500 dollars, and over 1000 dollars, respectively. Find $P(A)$ and $P(B)$.

(3) Are A and B independent events? (8)

Or

(b) (i) A box contains tags marked $1, 2, \ldots n$. (1) Two tags are chosen at random without replacement. Find the probability that the numbers on the tags will be consecutive integers. (2) Two tags are chosen at random with replacement. Find the probability that the numbers on the tags will be consecutive integers. (8)

(ii) Experience has shown that while walking in a certain park, the time X (in mins.), between seeing two people smoking has a density function of the form $f(x) = \begin{cases} \lambda x e^{-x}; & x > 0 \\ 0 & \text{otherwise} \end{cases}$

(1) Calculate the value of λ.

(2) Find the distribution function of X.

(3) What is the prob. that Jeff, who has just seen a person smoking, will see another person smoking in 2 to 5 minutes? In at least 7 minutes? (8)

12. (a) (i) The atoms of radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, what is the probability that during the next second the number of alpha particles emitted from 1 gram is

(1) at most 6 (2) at least 2 (3) at least 3 and at most 6? (8)

(ii) Starting at 5.00 a.m. every half hour there is a flight from San Francisco airport to Los Angeles International Airport. Suppose that none of these planes is completely sold out and that they always have room for passengers. A person who wants to fly to L.A. arrives at the airport at a random time between 8.45 a.m. and 9.45 a.m. Find the probability that she waits (1) at most 10 mins. (2) at least 15 mins. (8)

Or

(b) (i) X is normally distributed and the mean of X is 12 and S.D. is 4. Find the probability of the following :

(1) $X \geq 20$

(2) $0 \leq X \leq 12$

(3) Find x', when $P(X > x') = 0.24$

(4) Find x_0^1 and x_1^1, when $P(x_0^1 < X < x_1^1) = 0.50$ and $P(X > x_1^1) = 0.25$. (8)

(ii) A man with 'n' keys wants to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials required to open the door if unsuccessful keys are not eliminated from further selection. (8)

13. (a) (i) The joint pdf of a two-dimensional $RV(X,Y)$ is given by $f(x,y) = xy^2 + \frac{x^2}{8}$; $0 \leq x \leq 2$, $0 \leq y \leq 1$. Compute :

(1) $P\left(X > 1 / Y < \frac{1}{2}\right)$ (2) $P\left(Y < \frac{1}{2} / X > 1\right)$

(3) $P(X < Y)$ (4) $P(X + Y \leq 1)$. (8)

(ii) Find the coefficient of correlation for the following heights (in inches) of fathers (X) and their sons (Y) : (8)

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Or

(b) (i) The joint pmf of (X,Y) is given by $p(x,y) = k(2x + 3y)$, $x = 0,1,2$; $y = 1,2,3$. Find all the marginal probability distributions. Also find the probability distribution of $(X + Y)$. (10)

(ii) Can $Y = 5 + 2.8X$ and $X = 3 - 0.5Y$ be the estimated regression equations of Y on X and X on Y respectively? Explain your answer with suitable theoretical arguments. (6)

14. (a) (i) Show that the random process $X(t) = A\cos(w_0 t + \theta)$ is wide-sense stationary, if A and w_0 are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follows a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted. (8)

Or

(b) (i) (1) Define stationary transition probabilities. (3)

(2) Derive the Chapman-Kolmogorov equations for discrete-time Markov chain. (5)

(ii) On a given day, a retired English professor, Dr. Charles Fish, amuses himself with only one of the following activities : reading (activity 1), gardening (activity 2), or working on his book about a river valley (activity 3). For $1 \le i \le 3$, let $X_n = i$ if Dr. Fish devotes day 'n' to activity i. Suppose that $\{X_n;\, n = 1, 2, \ldots\}$ is a Markov chain, and depending on which of these activities on the next day is given by the t.p.m.

$$P = \begin{pmatrix} 0.30 & 0.25 & 0.45 \\ 0.40 & 0.10 & 0.50 \\ 0.25 & 0.40 & 0.35 \end{pmatrix}$$

Find the proportion of days Dr. Fish devotes to each activity. (8)

15. (a) (i) For the steady state M/M/1 queueing model, Prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n P_o$. (8)

(ii) On every Sunday morning, a Dental hospital renders free dental service to the patients. As per the hospital rules, 3 dentists who are equally qualified and experienced will be on duty then. It takes on an average 10 mins for a patient to get treatment and the actual time taken is known to vary approximately exponentially around this average. The patients arrive according to the Poisson distribution with an average of 12 per hour. The hospital management wants to investigate the following :

(1) The expected number of patients waiting in the queue.

(2) The average time that a patient spends at the hospital. (8)

Or

(b) (i) Self-Service system is followed in a super market at a metropolis. The customer arrivals occur according to a Poisson distribution with mean 40 per hour. Service time per customer is exponentially distributed with mean 6 mins.

(1) Find the expected number of customers in the system.

(2) What is the percentage of time that the facility is idle? (8)

(ii) Derive the Pollaczek-Khinchine formula for M/G/1 queueing model. (8)

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